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## XVIII.

## ON THE GROUP OF REAL LINEAR TRANSFORMATIONS WHOSE INVARIANT IS AN ALTERNATE BILINEAR FORM.

BY HENRY TABER.

Presented February 12, 1896.

LET  $G$  denote the group of linear automorphic transformations of the alternate bilinear form

$$\mathfrak{F} = \sum_{r=1}^{2n} \sum_{s=1}^{2n} (c_{rs} - c_{sr}) x_r y_s$$

with cogrediant variables and of non-zero determinant. On page 575 *et seq.*, Volume XLVI. of the *Mathematische Annalen*, I have shown that a transformation of group  $G$  can be generated by the repetition of an infinitesimal transformation of group  $G$  if, and only if, it is the second power of a transformation of group  $G$ . I now find, if  $\mathfrak{F}$  is real, that the same theorem holds for the sub-group of real transformations of group  $G$ . That is, if  $\mathfrak{F}$  is real, a real transformation of group  $G$  can be generated by the repetition of a real infinitesimal transformation of group  $G$  if, and only if, it is the second power of a real transformation of this group. Furthermore, if  $\mathfrak{F}$  is real, the second power of a real transformation of group  $G$  is the  $(2m)$ th power of a real transformation of this group for any even exponent  $2m$ .\*

If the transformation  $T$  is defined by the system of equations

$$x'_r = a_{r1}x_1 + a_{r2}x_2 + \dots + a_{r,2n}x_{2n} \quad (r = 1, 2, \dots, \overline{2n-1}, 2n),$$

let  $T_\lambda$  denote the transformation defined by the equations

$$x'_r = (a_{r1}x_1 + a_{r2}x_2 + \dots + a_{r,2n}x_{2n}) - \lambda x_r \quad (r = 1, 2, \dots, \overline{2n-1}, 2n),$$

$\lambda$  being a root of multiplicity  $m$  of the characteristic equation of  $T$ .

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\* For an odd exponent  $2m+1$ , any real transformation of group  $G$  is the  $(2m+1)$ th power of a real transformation of this group.

The nullity\* of  $T_\lambda$  is then at least one, and the nullity of successive powers of  $T_\lambda$  increases until a power of exponent  $\mu \equiv m$  is attained whose nullity is equal to  $m$ . The nullity of the  $(\mu + 1)$ th and higher powers of  $T_\lambda$  is then also  $m$ . If we designate respectively by

$$m_1, m_2, \dots, m_{\mu-1}, m_\mu = m,$$

the nullities of

$$T_\lambda, T_\lambda^2, \dots, T_\lambda^{\mu-1}, T_\lambda^\mu$$

then

$$m_1 \equiv m_2 - m_1 \equiv \dots \equiv m_\mu - m_{\mu-1} \equiv 1.$$

The numbers  $\mu_1, \mu_2$ , etc., may be termed the numbers *belonging* to the root  $\lambda$  of the characteristic equation of  $T$ .

If now  $T$  is the second power of a real transformation of group  $G$ , the numbers belonging to each negative root of the characteristic equation of  $T$  are all even. These conditions are probably not only necessary but sufficient in order that a real transformation  $T$  of group  $G$  may be the second power of a real transformation of this group.

\* The *nullity* of the transformation defined by the system of equations

$$x'_r = a_{r1}x_1 + a_{r2}x_2 + \dots + a_{rN}x_N \quad r = (1, 2, \dots, N),$$

is  $m$  if all the  $(m-1)$ th minors (the minor determinants of order  $N-m+1$ ) are zero, but not all the  $m$ th minors (the minor determinants of order  $N-m$ ) of the matrix.

$$\begin{array}{c} a_{11}, a_{12}, \dots \\ a_{21}, a_{22}, \dots \\ \dots \dots \dots \end{array}$$